3 The Cabling

At the MHz frequencies involved in NDE tests, the electrical cables that transfer the electrical pulses from the pulser to the sending transducer and from the receiving transducer to the receiver do not just pass those signals unchanged. Thus, significant cabling effects may be present in some ultrasonic testing setups. Here we will discuss models and measurements that can help us to quantitatively determine the effects of the cables. These models and measurements will enable us to predict how the voltage and current change from one end of the cable to the other (Fig. 3.1).

3.1 Cable Modeling

At the most fundamental level we can model a cable as a set of coaxial conductors transferring electrical and magnetic fields ($\mathbf{E}, \mathbf{H}$) from one end of the cable to the other, as shown in Fig. 3.2. It is shown in many texts on electromagnetism [3.1-3.7] that the fields at each end of the cable are related by the reciprocity relationship

$$\begin{aligned} &\int_{S_1} \left(\mathbf{E}_1^{(2)} \times \mathbf{H}_1^{(1)} - \mathbf{E}_1^{(1)} \times \mathbf{H}_1^{(2)} \right) \cdot \mathbf{n}_1 \, dS \\ &= \int_{S_2} \left(\mathbf{E}_2^{(2)} \times \mathbf{H}_2^{(1)} - \mathbf{E}_2^{(1)} \times \mathbf{H}_2^{(2)} \right) \cdot \mathbf{n}_2 \, dS, \end{aligned} \tag{3.1}$$

where ($\mathbf{E}_1, \mathbf{H}_1$) are fields at the left end of the cable acting over an area S_1 whose unit normal (pointing out from the cable) is $\mathbf{n}_1$, and ($\mathbf{E}_2, \mathbf{H}_2$) are the corresponding fields at the other end, S_2, whose outward normal is $\mathbf{n}_2$ as shown in Fig. 3.2. The superscripts (1) and (2) on the field variables in Eq. (3.1) designate these fields when the cable is under two different driving/termination conditions at its ends. These two driving/termination conditions are labeled as states (1) and (2). If the fields are carried in the cable as a fundamental propagating electromagnetic wave mode called a *TEM mode*, then it can be shown that the electric field, **E**, can be expressed in terms of a potential (voltage), V, across the two conductors in the cable

Fig. 3.1. A cable and the voltages and currents at its end connectors.

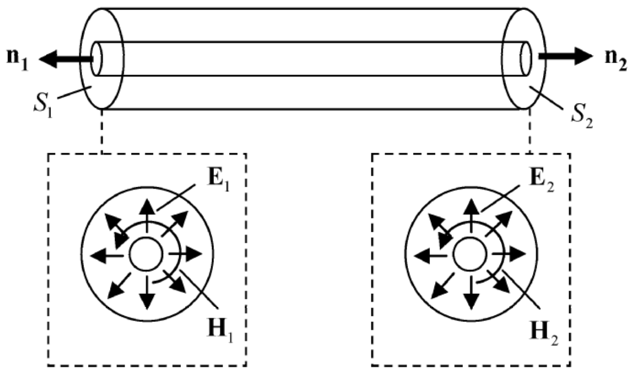

Fig. 3.2. The electrical and magnetic fields at the ends of a coaxial cable.

and the magnetic field, **H**, can be related to the current, I, flowing through the central conductor [3.4]. These relations are

$$\mathbf{E} = -\nabla V$$
$$I = \int_c \mathbf{H} \cdot d\mathbf{l}, \tag{3.2}$$

where c is a closed path taken around the central conductor of the cable and $d\mathbf{l}$ is a vector differential element along that path.

For such a propagating TEM mode it can also be shown that the reciprocity relationship of Eq. (3.1) reduces to a similar reciprocity relationship between the voltages and the currents in states (1) and (2) given by

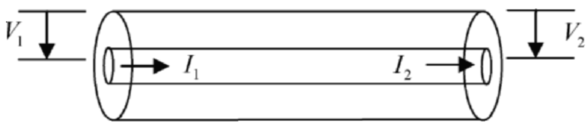

Fig. 3.3. A cable modeled in terms of the voltages and currents at its two ends (ports).

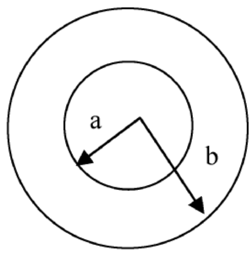

Fig. 3.4. Cross-section of an ideal circular coaxial cable where the radius of the inner conductor is *a* and the radius of the outer conductor is *b*.

$$V_1^{(1)}I_1^{(2)} - V_1^{(2)}I_1^{(1)} = V_2^{(1)}I_2^{(2)} - V_2^{(2)}I_2^{(1)} \tag{3.3}$$

so that we can then consider our cable as modeled in terms of these voltages and currents where I_1 is the current flowing into the cable at the left end and I_2 is the current flowing out of the cable at the other end (see Fig. 3.3). If the reciprocity relationship of Eq. (3.3) is satisfied for any set of driving/termination conditions, then it can also be shown that the voltage and current at one end (port) of the cable are linearly related to the voltage and current at the other end (port) and we can model the cable as a reciprocal two port system (see Appendix C) where one has

$$\begin{Bmatrix} V_1 \\ I_1 \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{Bmatrix} V_2 \\ I_2 \end{Bmatrix} \tag{3.4}$$

and $\det[\mathbf{T}] = T_{11}T_{22} - T_{12}T_{21} = 1$.

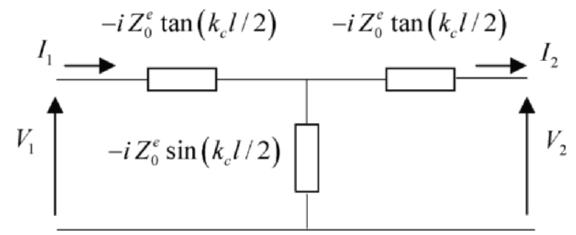

Fig. 3.5. An equivalent circuit model of a cable.

As developed in many electrical engineering texts, one can use a simple transmission line model of the cable and obtain an explicit expression for this transfer matrix $[\mathbf{T}]$ in the form [3.5]

$$\begin{Bmatrix} V_1 \\ I_1 \end{Bmatrix} = \begin{bmatrix} \cos(k_c l) & -iZ_0^e\sin(k_c l) \\ -i\sin(k_c l)/Z_0^e & \cos(k_c l) \end{bmatrix} \begin{Bmatrix} V_2 \\ I_2 \end{Bmatrix}, \tag{3.5}$$

where l is the length of the cable, Z_0^e is the characteristic impedance of the cable (in ohms), and $k_c = \omega / c$ is the wave number and c is the wave speed of signals in the cable. For an ideal circular coaxial cable as shown in Fig. 3.4 where the inner conductor is of radius a and the outer conductor is of radius b the characteristic impedance of the cable is given by [3.5]

$$Z_0^e = \frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon}}\ln\left(\frac{b}{a}\right), \tag{3.6}$$

where μ is the permeability and ε the permittivity of the material in the cable between the inner and outer conductors.

In Appendix C we showed how a simple RC circuit could be expressed in transfer matrix form as a two port system. Thus it is not surprising that conversely a two port system can also be expressed as an equivalent circuit. There are actually many different equivalent circuits that yield the same results as the transfer matrix. Figure 3.5 shows one commonly used circuit [3.1] that uses three impedances arranged in a T-shape to model the cable.

If our cable model is terminated with a impedance, Z_2^e, as shown in Fig. 3.6 (a) then the cable and its termination can be represented as a single equivalent impedance, Z_1^e, as shown in Fig. 3.6 (b). The behavior of

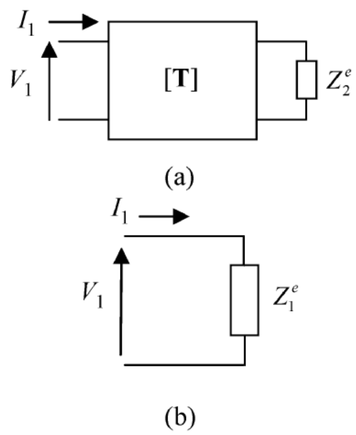

Fig. 3.6. (a) A cable terminated with an impedance, Z_2^e, and **(b)** the equivalent impedance, Z_1^e, of this terminated cable.

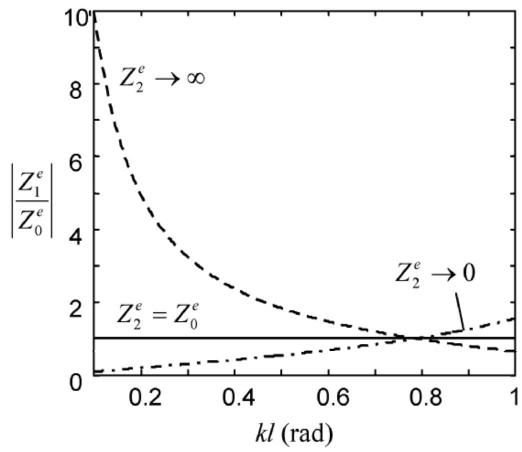

Fig. 3.7. The effect of different termination conditions on the equivalent impedance of a cable.

this equivalent impedance versus the non-dimension frequency $k_c l$ is shown in Fig. 3.7 for open-circuit ($Z_2^e \to \infty$) termination, short-circuit ($Z_2^e = 0$) termination, and termination at the characteristic impedance of

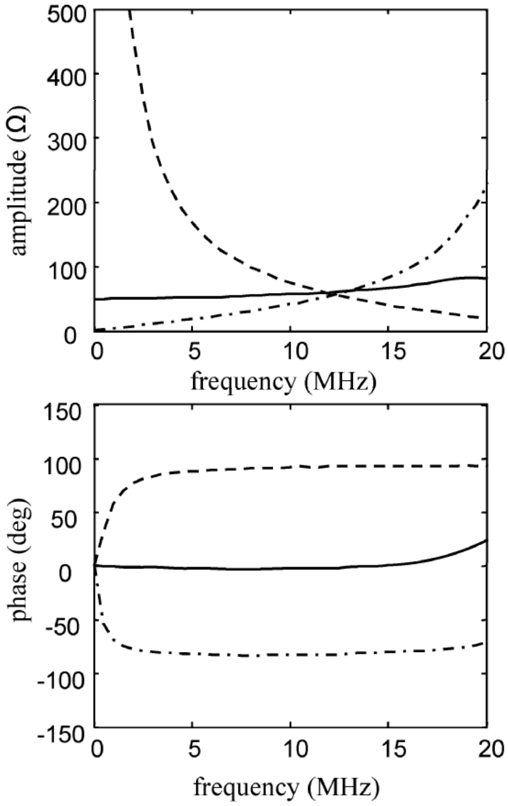

Fig. 3.8. Measured values of the magnitude and phase of a 50 ohm cable under open-circuit (dashed line), short-circuit (dashed-dotted line), and 50 ohm (solid line) termination conditions.

the cable ($Z_2^e = Z_0^e$). It can be seen that the open- and short-circuit cases generate frequency dependent equivalent impedances while in the matched termination case the equivalent impedance is frequency independent. This same behavior is seen when the equivalent impedance of a 50 ohm cable is measured experimentally, as shown in Fig. 3.8. The cables used in an ultrasonic test for sound generation and reception are terminated/driven by ultrasonic transducers which in general are not matched in impedance to the cable so that inherently we can expect some frequency dependent effects due to the cabling in NDE tests.

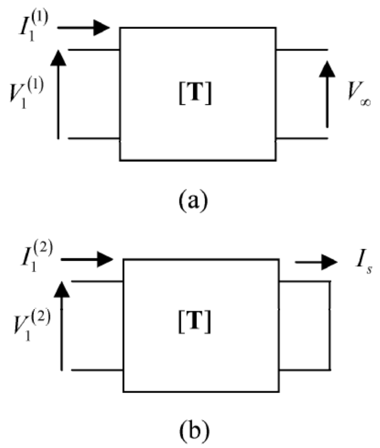

Fig. 3.9. (a) A cable, modeled as a two port system, under open-circuit conditions, and **(b)** under short-circuit conditions. Measurements of the voltages and currents shown can be used to determine the transfer matrix of the cable.

3.2 Measurement of the Cabling Transfer Matrix

As can be seen from Figs. 3.7 and 3.8 a simple two port model can accurately represent the behavior of an ordinary coaxial cable. However, we do not ordinarily know all the detailed parameters that are needed to obtain the transfer matrix components in Eq. (3.5). Furthermore, in immersion NDE testing, such cabling is connected to fixtures that support the transducer in an immersion tank and the details of the cabling within the fixtures are in general also not known. This is not a problem since it is possible to directly measure the transfer matrix components of the combined cabling and fixtures in situ by attaching the cable/fixture to a driving source, such as the ultrasonic pulser, and making a series of voltage and current measurements under different cable/fixture termination conditions. Figure 3.9 (a) shows a two port model of a cable under open-circuit conditions at its output port and driving voltage $V_1^{(1)}$ and current $I_1^{(1)}$ at its input port while Fig. 3.9 (b) shows the same model under short-circuit conditions at the output port with driving voltage $V_1^{(2)}$ and current $I_1^{(2)}$ at the input port. From Eq. (3.4) it is easy to see that:

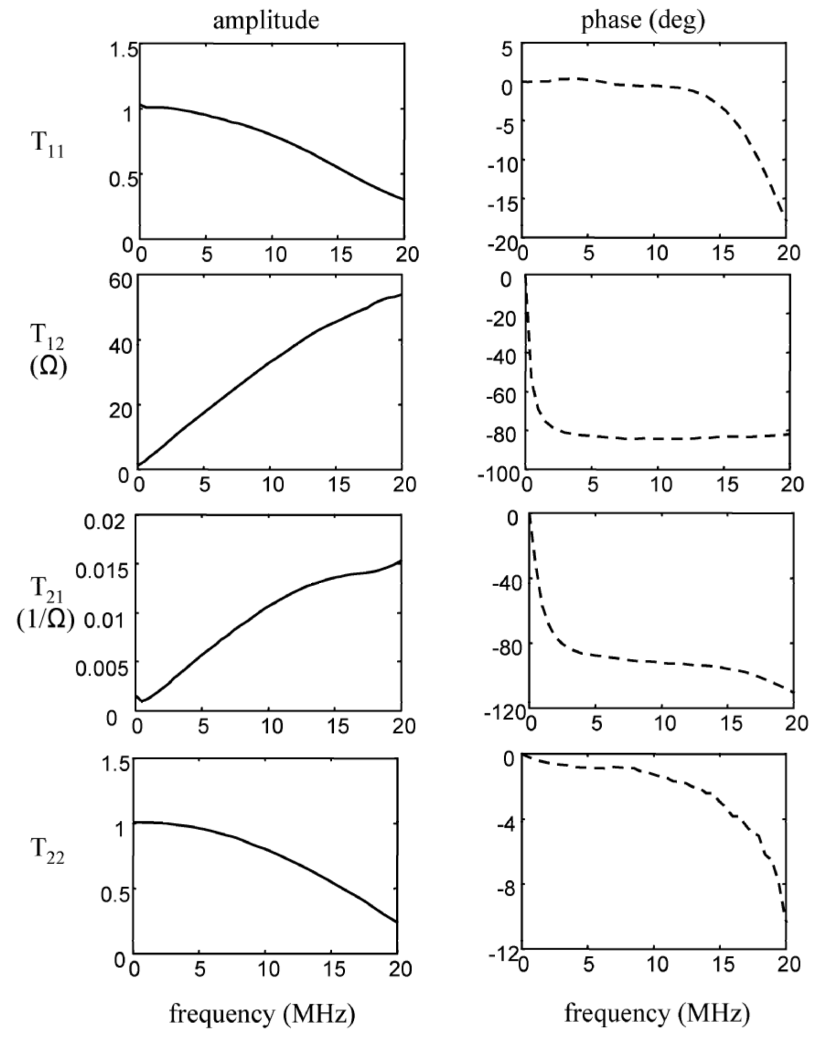

Fig. 3.10. Measured values of the magnitudes and phases of the transfer matrix components versus frequency for a cable.

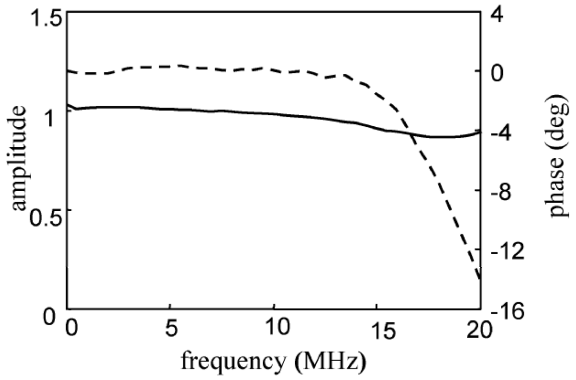

Fig. 3.11. A check on the satisfaction of reciprocity ($\det[\mathbf{T}]=1$) for the measured transfer matrix components of Fig. 3.10. The amplitude (solid line) and phase (dashed line) of the determinant are shown.

$$\begin{aligned}
T_{11}(\omega) &= V_1^{(1)}(\omega)/V_\infty(\omega) \\
T_{21}(\omega) &= I_1^{(1)}(\omega)/V_\infty(\omega) \\
T_{12}(\omega) &= V_1^{(2)}(\omega)/I_s(\omega) \\
T_{22}(\omega) &= I_1^{(2)}(\omega)/I_s(\omega)
\end{aligned} \tag{3.7}$$

so that all the transfer matrix elements can be obtained by making measurements of the voltages and currents in these two states: $v_1^{(m)}(t), i_1^{(m)}(t)$, $v_\infty(t)$, $i_s(t)$ ($m=1{,}2$) and Fourier transforming them to obtain $V_1^{(m)}(\omega), I_1^{(m)}(\omega)$, $V_\infty(\omega), I_s(\omega)$ ($m=1{,}2$). The consistency of these measured transfer matrix elements can be checked by the reciprocity relationship $T_{11}T_{22}-T_{12}T_{21}=1$.

Figure 3.10 shows the transfer matrix components found in this manner as a function of frequency for a cable (both amplitude and phase are plotted). It can be seen that the measured magnitudes of these components do exhibit the cosine and sine function behavior of Eq. (3.5) and the measured phase terms also generally follow that simple model behavior. As a reciprocity check on these measurements we can compute the determinant of the measured transfer matrix. Figure 3.11 shows that det[**T**] = 1 is well satisfied over a wide range of frequencies.

3.3 References

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3.4 Exercises

1. Consider a 1 meter long, 50 ohm cable, where the wave speed in the cable is one half the wave speed of light, c_0 ,in a vacuum (c_0 = 2.998 x 10^8 m/sec). Determine the transfer matrix components of the cable at 10 kHz, 100 kHz, 1 MHz, 20 MHz.

2. Consider a cable for which we wish to measure the transfer matrix components (as a function of frequency). We can do this in MATLAB for a function cable_X which has the calling sequence:

```
>> [ v1, i1, vt, it] = cable_X( V, dt, R,  L, 'term');
```

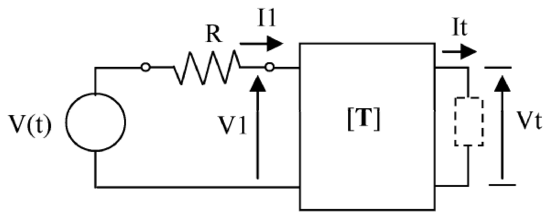

Fig. 3.12. A measurement setup for obtaining the transfer matrix components of a cable.

The input arguments of cable_X are as follows. V is a sampled voltage source versus time, where the sampling interval is dt. R is an external resistance (in ohms). This source and resistance are connected in series to one end of the cable, which is of length L (in m) as shown in Fig. 3.12. The other end of cable can be either open-circuited or short-circuited. The string ‘term’ specifies the termination conditions. It can be either ‘oc’ for open-circuit or ‘sc’ for short-circuit. Cable_X then returns the “measured” sampled voltages and currents versus time (v1, i1, vt, it) where (v1, i1) are on the input side of the cable and (vt, it) are at the terminated end (Note: for open-circuit conditions it = 0 and for short-circuit conditions vt = 0). As a voltage source to supply the V input to cable_X use the MATLAB function pulserVT. For a set of sampled times this function returns a sampled voltage output that is typical of a “spike” pulser. Make a vector, t, of 512 sampled times ranging from 0 to 5 μsec with the MATLAB call:

```
>> t = s_space(0, 5, 512);
```

(see the discussion of the s_space function in Appendix A; a code listing of the function is given in Appendix G) and call the pulserVT function as follows:

```
>> V = pulserVT(200, 0.05, 0.2, 12, t);
```

For the resistance, take R = 200 ohms, and specify the length of the cable as L = 2 m.

Using Eq. (3.7), determine the four cable transfer matrix components and plot their magnitudes and phases from 0 to 30 MHz. Note that the outputs of cable_X are all time domain signals but the quantities in Eq. (3.7) are all in the frequency domain so you will need to define a set of 512 sampled frequency values, f, through:

```
>> dt = t(2) –t(1);
>> f =s_space(0, 1/dt, 512);
```

What is the range of frequencies contained in f here?